where

$$\begin{bmatrix} T \\ \hline H \end{bmatrix}^{-1} = [A \mid B] \quad \text{and} \quad AT + BH = i$$

$$\begin{bmatrix} T \\ \hline H \end{bmatrix} [A \mid B] = I$$

should be satisfied. Then the differential equation for the observer will be given by

$$\dot{\hat{y}}(t) = TFA\hat{y}(t) + TFBz(t) + TLu(t) \tag{8}$$

and the state estimates are generated by

$$\hat{x}(t) = A\hat{y}(t) + Bz(t) \tag{9}$$

The observer error has exponential convergence rate[2] and is a function of the choice of B. It can be shown that if the system is completely observable, the matrix B can be chosen to produce any desired response by selecting the eigenvalues properly. It is known (see Refs. 2 and 3) that A and T are not unique in the time-invariant actuation and that the admissible pair (A,T) defines an equivalent class in the class of observers with exponential error convergence rates. It should be noted that there is no rule that directs one in selecting the desired

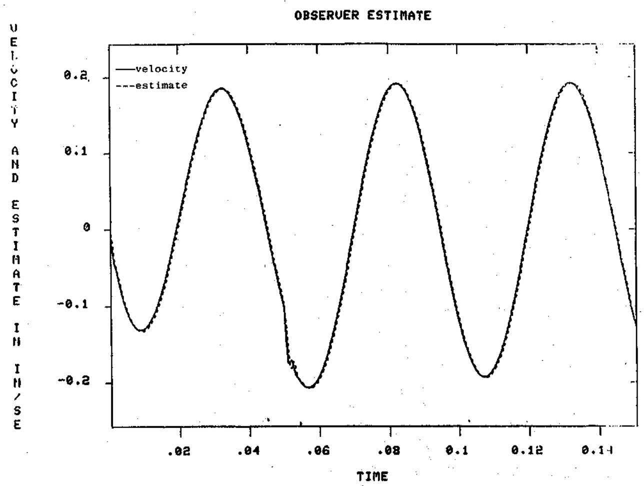

Fig. 1 Observer estimate of perturbation in velocity.

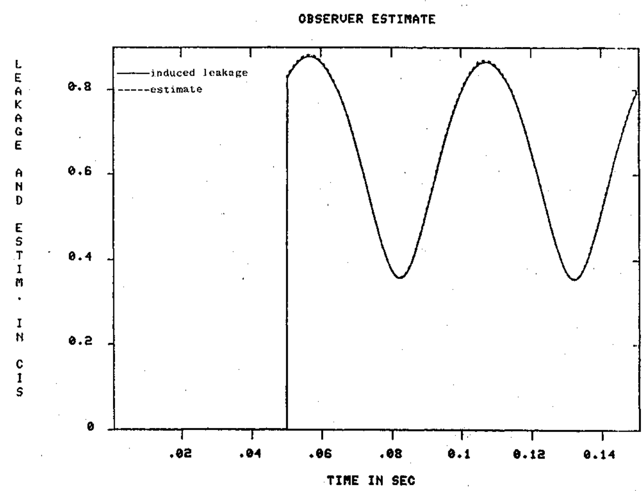

Fig. 2 Observer estimate of induced leakage.

eigenvalue. One should consider the specific problem at hand and make a choice based on engineering intuition and practical considerations.

III. Simulation and Results

The main objective of the simulations in the present study was to assess the applicability of observers for state and parameter estimation of hydromechanical servoactuation systems. The results are very promising. The system considered was generic, and its observability, especially for the reduced-order observer situation, proves to be practicably significant.

The velocity and acceleration of the actuator piston rod was estimated herein, as well as leakage across the piston. However, various other parameters, such as spool position of the servovalve, the load pressure, etc., can be estimated in a similar manner. The results are plotted in Figs. 1 and 2. A small leakage was introduced through a small opening on the piston seal, at time = .05 s. The resulting leakage is seen in Fig. 2 and the perturbation in velocity in Fig. 1.

Even though it is theoretically possible to estimate exactly the abovementioned parameters, practical considerations vis-a-vis the selection of eigenvalues for observer performance result in a slight estimation error.

IV. Conclusions

Reduced-order observer theory has been successfully applied to hydromechanical servoactuators. States and a parameter of the actuator have been estimated with minimal estimation error. This could lead to significant developments in fluid power control in that it opens the door to backup provisions, as well as to performance enhancement and failure detection of actuation systems. The measurement system chosen was realistic, and thus successful practical implementation of such a scheme seems almost certain.

References

[1]Leonard, J.B., "A System Look at Electromechanical Actuation for Primary Flight Control," *Proceedings of the IEEE 1983 National Aerospace and Electronics Conference (NAECON)*, pp. 80-86.

[2]Kwa Kernaak, H. and Sivan, R., *Linear Optimal Control Systems*, Wiley Interscience, New York, 1972.

[3]Viersma, T.J., *Analysis Synthesis and Design of Hydraulic Servomechanism and Pipelines*, Elsevier Scientific Publishing Co., 1980.

[4]Merritt, H.E., *Hydraulic Control Systems*, John Wiley & Sons, Inc., New York, 1966.

[5]Gelb, A. (ed.), *Applied Optimal Estimation*, MIT Press, Cambridge, MA, 1974.

[6]Lilly, J.H., "A General Adaptive Observer Structure Subject to Unmodeled Dynamics," *Proceedings of the 1985 American Control Conference*, 1985.

A Computational Method to Solve Nonautonomous Matrix Riccati Equations

M.B. Subrahmanyam*

University of Missouri-Columbia, Columbia, Missouri

Introduction

WE propose a method to solve for the Riccati matrix, which is effective even when the solution of the Riccati equation becomes unbounded at a finite number of points.

Received June 14, 1985; revision received Sept. 24, 1985.

*Assistant Professor, Department of Electrical and Computer Engineering.

Any integration scheme of the Riccati equation would stop at the singularities.

Consider the matrix Riccati equation

$$-\dot{P}(t)=P(t)A(t)+A^T(t)P(t)-P(t)B(t)P(t)+C(t) \quad (1)$$

on the time interval $[t_0, t_f]$ with either the initial condition

$$P(t_0)=P_0 \quad (2)$$

or the terminal condition

$$P(t_f)=P_f \quad (3)$$

It is well known[1] that

$$\begin{bmatrix} \dot{x} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} A(t) & -B(t) \\ -C(t) & -A^T(t) \end{bmatrix} \begin{bmatrix} x \\ \psi \end{bmatrix} \quad (4a)$$

with either

$$\begin{bmatrix} x(t_0) \\ \psi(t_0) \end{bmatrix} = \begin{bmatrix} x_0 \\ P_0 x_0 \end{bmatrix} \quad (4b)$$

or

$$\begin{bmatrix} x(t_0) \\ \psi(t_f) \end{bmatrix} = \begin{bmatrix} x_0 \\ P_f x_f \end{bmatrix} \quad (4c)$$

gives rise to the relation

$$\psi(t)=P(t)\,x(t) \quad (5)$$

The aim is to find $P(t)$ numerically from Eq. (4). All of the analysis applies also to the Lyapunov equation by setting $B(t)=0$.

Method of Solution

Let

$$Y(t)=\begin{bmatrix} x(t) \\ \psi(t) \end{bmatrix}, \quad D(t)=\begin{bmatrix} A(t) & -B(t) \\ -C(t) & -A^T(t) \end{bmatrix} \quad (6)$$

Then Eq. (4a) becomes

$$\dot{Y}(t)=D(t)Y(t) \quad (7)$$

with the associated initial or boundary condition.

We subdivide $[t_0,t_f]$ into N equal intervals of length Δt. Let $t_i=t_{0+i}\Delta t$ and $t_{i+½}=t_i+\Delta t/2$. We discretize Eq. (6) by the scheme

$$\tilde{Y}(t_{i+1})=\sum_{k=0}^{K} D^k(t_{i+½})\frac{\Delta t^k}{k!}\tilde{Y}(t_i) \quad (8)$$

where K will be chosen to be 2 for time-varying systems. The reason for this is contained in Eq. (11) of the following lemma. This, of course, would make the method extremely fast.

Lemma: Assume that $D(t)$ is twice continuously differentiable in t. Let $\tilde{Y}(t_{i+1})$ be given by Eq. (8) and $Y(t_{i+1})$ be the true value at t_{i+1} with $\tilde{Y}(t_i)=Y(t_i)$. Then

$$\|\tilde{Y}(t_{i+1})-Y(t_{i+1})\|=O(\Delta t^3) \quad (9)$$

Proof: We have, by Taylor's series,

$$\begin{aligned} Y(t_{i+1}) &= Y(t_i)+\dot{Y}(t_i)\Delta t+\ddot{Y}(t_i)\frac{\Delta t^2}{2!}+O(\Delta t^3) \\ &= Y(t_i)+D(t_i)Y(t_i)\Delta t+(D(t_i)\dot{Y}(t_i) \\ &\quad +\dot{D}(t_i)Y(t_i))\frac{\Delta t^2}{2!}+O(\Delta t^3) \\ &= Y(t_i)+\left(D(t_i)+\dot{D}(t_i)\frac{\Delta t}{2}\right)Y(t_i)\Delta t \\ &\quad +D^2(t_i)Y(t_i)\frac{\Delta t^2}{2!}O(\Delta t^3) \end{aligned}$$

Since

$$D(t_{i+½})=D(t_i)+\dot{D}(t_i)\frac{\Delta t}{2}+O(\Delta t^2)$$

we get

$$Y(t_{i+1})=Y(t_i)+D(t_{i+½})Y(t_i)\Delta t+D^2(t_i)Y(t_i)\frac{\Delta t^2}{2!}+O(\Delta t^3) \quad (10)$$

From Eq. (8),

$$\tilde{Y}(t_{i+1})=Y(t_i)+D(t_{i+½})Y(t_i)\Delta t +D^2(t_{i+½})Y(t_i)\frac{\Delta t^2}{2!}+O(\Delta t^3) \quad (11)$$

From Eqs. (10) and (11), we have

$$\|Y(t_{i+1})-\tilde{Y}(t_{i+1})\| \le \|D^2(t_i)-D^2(t_{i+½})\| \; \|Y(t_i)\|\frac{\Delta t^2}{2!}+O(\Delta t^3) \quad (12)$$

Since $\|D^2(t_i)-D^2(t_{i+½})\|=O(\Delta t)$, the proof of the lemma is complete.

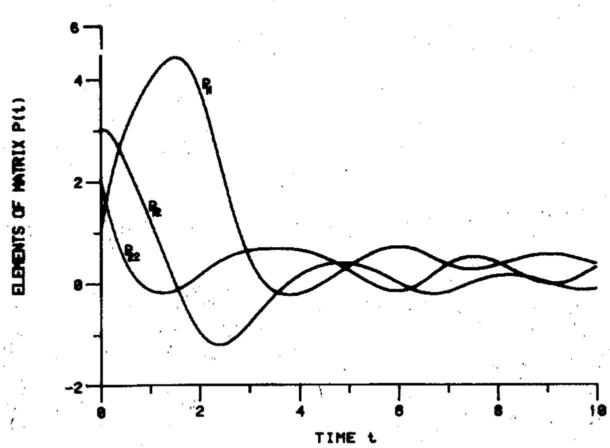

Fig. 1 Elements of matrix P vs t for Example 1.

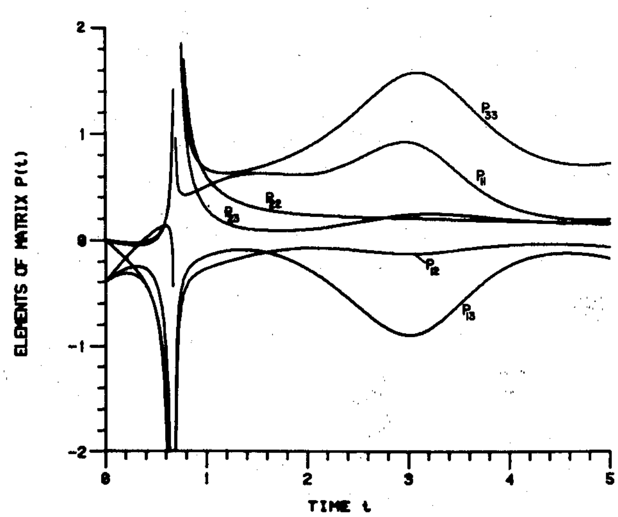

Fig. 2 Elements of matrix P vs t for Example 2.

Note that, for time-invariant systems, Eq. (8) is exact except for the truncation error, which is of order $O(\Delta t^{K+1})$.

We now find the recursive relation for $P(t)$. Let

$$G_i = \sum_{k=0}^{K_i} D^k (t_{i+1/2}) \frac{\Delta t^k}{k!} \tag{13}$$

From Eq. (8), we have

$$\begin{bmatrix} x(t_{i+1}) \\ \psi(t_{i+1}) \end{bmatrix} = \begin{bmatrix} G_i^{11} & G_i^{12} \\ G_i^{21} & G_i^{22} \end{bmatrix} \begin{bmatrix} x(t_i) \\ \psi(t_i) \end{bmatrix} \tag{14}$$

It follows from Eqs. (14) and (5) that

$$P(t_{i+1}) = [G_i^{21} + G_i^{22} P(t_i)][G_i^{11} + G_i^{12} P(t_i)]^{-1} \tag{15}$$

or

$$P(t_i) = -[G_i^{22} - P(t_{i+1}) G_i^{12}]^{-1} [G_i^{21} - P(t_{i+1}) G_i^{11}] \tag{16}$$

Equations (15) and (16) are suitable for initial and terminal conditions (2) and (3), respectively. Equation (15) is to be used with Kalman filtering and Eq. (16) with quadratic regulation. Note that, because of the structure of Eqs. (15) and (16), $P(t)$ will be computed on all of $[t_0, t_f]$ irrespective of the presence of singularities in the solution. This is because $P(t)$ may assume large computed values near the singularities but will not exceed the floating point limit of the computer.

In order to control the size of G_i given by Eq. (13), we could normalize G_i such that its maximal element is one without affecting relations (15) and (16).

Examples

All the computations for the examples were performed on a VAX 11/730 in double precision with $\Delta t = 0.01$.

Example 1. With

$$A = \begin{bmatrix} 0 & 1/2 \\ -1 & \sqrt{2}/5 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 \\ 0 & e^{-t} - 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 \\ 0 & -1/4 - 1/2 \sin 2t \end{bmatrix} \tag{17}$$

and

$$P(0) = \begin{bmatrix} 1 & 3 \\ 3 & 2 \end{bmatrix} \tag{18}$$

the solution is computed using (15) and shown in Fig. 1. The result is identical to Fig. 12 of Ref. 2.

Example 2. Consider an example from Ref. 2 with a singular solution. In this case,

$$A = \begin{bmatrix} 1/2 & -1 & 0 \\ 1 & 1/2 & -1/2 \cos 2t \\ -1/2 \sin 2t & -1 & 0 \end{bmatrix}$$

$$B = -\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 + 1/2 \sin 2t \end{bmatrix}$$

$$C = \begin{bmatrix} -e^{-t/2} & 0 & 0 \\ 0 & -e^{-t/2} & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{19}$$

with initial condition $P(0) = -0.4\, I$. The results are shown in Fig. 2. Note that this figure is very similar to Fig. 13 of Ref. 2, where nonlinear superposition formulas are used to solve the matrix Riccati equations. In this case we have a singularity at $t = 0.66$.

These two examples have time-varying elements. The procedure for time-invariant systems is similar and much more accuracy can be expected in the steady-state solution for this case by making K larger in Eq.(8).

References

[1]Kwakernaak, H. and Sivan, R., *Linear Optimal Control Systems*, Wiley-Interscience, New York, 1972, pp. 201-220.

[2]Rand, D. W. and Winternitz, P., "Nonlinear Superposition Principles: A New Numerical Method for Solving Matrix Riccati Equations," *Computer Physics Communications*, Vol. 33, Oct. 1984, pp. 305-328.

Roll Motion of a Wraparound Fin Configuration at Subsonic and Transonic Mach Numbers

Young Hoon Kim*
Agency of Defense Development
Republic of Korea
and
G.L. Winchenbach†
Air Force Armament Laboratory
Eglin Air Force Base, Florida

Nomenclature

A = reference area
C_ℓ = total roll moment coefficient, $\ell / \frac{1}{2} \rho V^2 A d$
$C_{\ell_{p+}}, C_{\ell_{p-}}$ = roll damping derivatives in the clockwise direction, $\partial C_\ell / \partial (p_+ d/2V)$, and counterclockwise direction, $\partial C_\ell / \partial (p_- d/2V)$, respectively
$C_{\ell_{pV}}$ = variation of roll deceleration due to velocity change
C_{ℓ_0} = total roll driving moment coefficient
C_{ℓ_δ} = roll moment coefficient due to fin cant
$C_{\ell_{0V}}, C_{\ell_{0V^2}}$ = variations of roll driving moment due to linear and squared velocity changes, respectively
$C_{\ell_{\gamma\bar{\alpha}^2}}$ = induced roll moment derivative
d = reference diameter
I_x = axial moment of inertia
N = number of fins
p = missile spin rate
$\bar{q}$ = dynamic pressure, $\rho V^2/2$
V = total missile velocity
V_{ref} = reference velocity
$\bar{\alpha}$ = missile total angle of attack, $\sin^{-1}[\sqrt{v^2 + w^2}/V]$
δ = fin cant angle
ξ = $\sin\bar{\alpha}$
γ = aerodynamic roll angle, $\tan^{-1}[v/w]$
ϕ = roll angle
ρ = air density

Submitted May 8, 1985; presented as Paper 85-1777 at the AIAA Atmospheric Flight Mechanics Conference, Snowmass, CO., Aug. 19-21, 1985; revision receiced Sept. 13, 1985.

*Exchange Scientist

†Chief, Aeroballistics Section.